Zbirka nalog za srednje šole: MATEMATIKA
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GEOMETRIJA V RAVNINI
Poglavje 20: Kotne funkcije ostrih kotov

Str. 106, naloga 729. Preoblikuj v enočlenik in poenostavi

a) $1-\sin^2 x+\cos^2 x$ b) $\sin x\, tgx\, ctg^2 x$ c) $\cos x+\sin x\, tgx$

č) $tgx+ctgx$ d) $\sin^{-1} x-\sin x$ e) $(1-\sin x)(1+\sin^{-1} x)$

f) $\dfrac{2\cos^2 x-\sin^2 x+1}{3\cos x}$ g) $(\sin x+\cos x)^2-(\sin x-\cos x)^2$ h) $\dfrac{tgx-\sin x\cos x}{\sin^2 x}$

i) $\dfrac{\sin x+\cos x\; tgx}{tgx}$ j) $\dfrac{ctgx+1}{\sin x+\cos x}$ k) $\dfrac{\sin x+2\sin x\cos x}{2+\cos x-2\sin^2 x}$

l) $\dfrac{\cos^{-1} x-\cos x}{\sin x}$ m) $\dfrac{1}{1+\sin x}+\dfrac{1}{1-\sin x}$ n) $\dfrac{1+\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x}$

o) $\dfrac{(1-\sin x)(1+\sin x)}{(1+\cos x)(1-\cos x)}$ p) $\sqrt{\dfrac{1+ctg^2 x}{1+tg^2 x}}$ r) $ctgx-\dfrac{\cos x}{\sin x-\sin^{-1} x}$

Razlaga:
Pri poenostvljanju izrazov s kotnimi funkcijami si pomagamo z zvezami:

(1) $\sin^2 x+\cos^2 x=1$ $\begin{cases}\sin^2 x=1-\cos^2 x & \text{(1a)}\\ \cos^2 x=1-\sin^2 x & \text{(1b)}\end{cases}$

(2) $tgx=\dfrac{\sin x}{\cos x}$ (3) $ctgx=\dfrac{\cos x}{\sin x}$ (4) $tgx\, ctgx=1$

(5) $1+tg^2x=\dfrac{1}{\cos^2 x}$ (6) $1+ctg^2x=\dfrac{1}{\sin^2 x}$ (7) $\sin 2x=2\sin x\cos x$

Enočlenik ima lahko samo koeficient, množenje in potenciranje.

Upoštevamo tudi dogovore: $\sin^2 x=(\sin x)^2$, $tgx=\tan x$, $ctgx=\cot anx$

Rešitve:

a) $1-\sin^2 x+\cos^2 x$ $=\cos^2 x+\cos^2 x$
$=2\cos^2 x$

b) $\sin x\, tgx\, ctg^2 x$ = (to je že enočlenik, vendar ga lahko še poenostavimo)

(2), (3) $=\sin x\dfrac{\sin x}{\cos x}\left(\dfrac{\cos x}{\sin x}\right)^2$

$=\sin x\dfrac{\sin x}{\cos x}\dfrac{\cos^2 x}{\sin^2 x}$

(okrajšam) $=\cos x$

c) $\cos x + \sin x\ tgx = \cos x + \sin x.\frac{\sin x}{\cos x}$ (2) $= \cos x + \frac{\sin^2 x}{\cos x}$ $= \frac{\cos^2 x + \sin^2 x}{\cos x}$ (1) $= \frac{1}{\cos x} = \cos^{-1} x$	č) $tgx + ctgx = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ (2), 3) (1) $= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ $= \frac{1}{\sin x \cos x}.\frac{2}{2}$ $= \frac{2}{2\sin x \cos x}$ (7) $= \frac{2}{\sin 2x}$ $= 2\sin^{-1}(2x)$
d) $\sin^{-1} x - \sin x = \frac{1}{\sin x} - \sin x$ $= \frac{1-\sin^2 x}{\sin x}$ (1b) $= \frac{\cos^2 x}{\sin x}$ (3) $= ctgx \cos x$	e) $(1-\sin x)(1+\sin^{-1} x) = (1-\sin x)(1+\frac{1}{\sin x})$ $= (1-\sin x)\frac{\sin x + 1}{\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{\sin x}$ $(a-b)(a+b) = a^2 - b^2$ $= \frac{1-\sin^2 x}{\sin x}$ (1b) $= \frac{\cos^2 x}{\sin x}$ (3) $= ctgx \cos x$
f) $\frac{2\cos^2 x - \sin^2 x + 1}{3\cos x} =$ $= \frac{2(1-\sin^2 x) - \sin^2 x + 1}{3\cos x}$ $= \frac{2 - 2\sin^2 x - \sin^2 x + 1}{3\cos x}$ $= \frac{3 - 3\sin^2 x}{3\cos x}$ (1b) $= \frac{3(1-\sin^2 x)}{3\cos x}$ $= \frac{\cos^2 x}{\cos x}$ $= \cos x$	g) $(\sin x + \cos x)^2 - (\sin x - \cos x)^2 =$ $= \sin^2 x + 2\sin x \cos x + \cos^2 x -$ $-(\sin^2 x - 2\sin x \cos x + \cos^2 x)$ (1) $= 1 + 2\sin x \cos x - (1 - 2\sin x \cos x)$ $= 1 + 2\sin x \cos x - 1 + 2\sin x \cos x$ (7) $= 2.2\sin x \cos x$ $2\sin 2x$

h) $\frac{tgx - \sin x\cos x}{\sin^2 x} = \frac{\frac{\sin x}{\cos x} - \sin x\cos x}{\sin^2 x} = \frac{\frac{\sin x - \sin x\cos^2 x}{\cos x}}{\frac{\sin^2 x}{1}}$ (1a) $= \frac{\sin x(1-\cos^2 x)}{\sin^2 x\cos x}$ $= \frac{\sin x\sin^2 x}{\sin^2 x\cos x}$ (2) $= \frac{\sin x}{\cos x} = \underline{tgx}$	i) $\frac{\sin x + \cos x\, tgx}{tgx} = \frac{\sin x + \cos x\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x}}$ (2) $= \frac{\sin x + \sin x}{\frac{\sin x}{\cos x}}$ $= \frac{2\sin x\cos x}{\sin x}$ $= \underline{2\cos x}$
j) $\frac{ctgx+1}{\sin x+\cos x} = \frac{\frac{\cos x}{\sin x}+1}{\sin x + cox}$ (3) $= \frac{\frac{\cos x+\sin x}{\sin x}}{\frac{\sin x+\cos x}{1}}$ $= \frac{\cos x+\sin x}{\sin x(\sin x+\cos x)}$ $= \frac{1}{\sin x} = \underline{\sin^{-1} x}$	k) $\frac{\sin x + 2\sin x\cos x}{2+\cos x - 2\sin^2 x} =$ $= \frac{\sin x(1+2\cos x)}{2+\cos x-2(1-\cos^2 x)}$ (1a) $= \frac{\sin x(1+2\cos x)}{2+\cos x-2+2\cos^2 x}$ $= \frac{\sin x(1+2\cos x)}{\cos x+2\cos^2 x}$ $= \frac{\sin x(1+2\cos x)}{\cos x(1+2\cos x)}$ (2) $= \frac{\sin x}{\cos x} = \underline{tgx}$
l) $\frac{\cos^{-1}x - \cos x}{\sin x} = \frac{\frac{1}{\cos x}-\cos x}{\sin x}$ (1a) $= \frac{\frac{1-\cos^2 x}{\cos x}}{\frac{\sin x}{1}}$ (2) $= \frac{\sin^2 x}{\sin x\cos x} = \frac{\sin x}{\cos x}$ $= \underline{tgx}$	m) $\frac{1}{1+\sin x}+\frac{1}{1-\sin x} = \frac{1-\sin x+1+\sin x}{(1+\sin x)(1-\sin x)}$ (1b) $= \frac{2}{1-\sin^2 x}$ $= \frac{2}{\cos^2 x}$ $= \underline{2\cos^{-2} x}$

n)	$\frac{1+\cos x}{\sin x}+\frac{\sin x}{1+\cos x}=$ $=\frac{(1+\cos x)^2+\sin^2 x}{\sin x(1+\cos x)}=\frac{1+2\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}=$ $=\frac{2+2\cos x}{\sin x(1+\cos x)}=\frac{2(1+\cos x)}{\sin x(1+\cos x)}=\frac{2}{\sin x}$ $=\underline{2\sin^{-1} x}$
o)	$\frac{(1-\sin x)(1+\sin x)}{(1+\cos x)(1-\cos x)}=\frac{1-\sin^2 x}{1-\cos^2 x}=\frac{\cos^2 x}{\sin^2 x}=\underline{ctg^2 x}$ (3)
p)	$\sqrt{\frac{1+ctg^2 x}{1+tg^2 x}}=\sqrt{\frac{\frac{1}{\sin^2 x}}{\frac{1}{\cos^2 x}}}$ (6), (5) $=\sqrt{\frac{\cos^2 x}{\sin^2 x}}=\frac{\cos x}{\sin x}=\underline{ctgx}$ (3)
r)	$ctgx-\frac{\cos x}{\sin x-\sin^{-1} x}=\frac{\cos x}{\sin x}-\frac{\cos x}{\sin x-\frac{1}{\sin x}}=\frac{\cos x}{\sin x}-\frac{\cos x}{\frac{\sin^2 x-1}{\sin x}}$ $=\frac{\cos x}{\sin x}-\frac{\sin x\cos x}{-(1-\sin^2 x)}=\frac{\cos x}{\sin x}+\frac{\sin x\cos x}{\cos^2 x}$ $=\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}=\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}=\frac{1}{\sin x\cos x}\cdot\frac{2}{2}=\frac{2}{\sin 2x}=$ $=\underline{2\sin^{-1}(2x)}$