Zbirka nalog za srednje šole: MATEMATIKA

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Poglavje VIII: Reševanje enačb; stran 35, naloge:

3.a) $\sin x = 0$ 3.f) $\sin x = \frac{\sqrt{3}}{2}$ 3.g) $\sin x = 1{,}5$ 4.a) $\sin x = \frac{1}{2}$

5.č) $\sin(2x-1) = \frac{1}{2}$ 5.d) $\sin(x+3) = 0{,}731$ 5.f) $\sin(x-90^0) + \sin(x-180^0) = 0$

6.d) $\sin^2 x - \cos^2 x = \frac{1}{2}$ 7.a) $\sin^2 x - 5\sin x - 6 = 0$

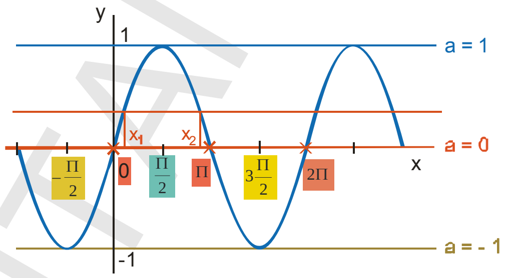
Razlaga:

Rešiti moram enačbo v obliki: sinx=a, kjer je $-1 \le a \le 1$

Posebej pogledamo naslednje vrednosti sinusa:

(Glej tudi nalogo iz tega učbenika: Poglavje IV.: Grafa funkcij sinus in kosinus, naloga 19j:

Narišem graf funkcije $y = \sin x$ in iščem presečišče s funkcijo $y = a$:



$$x_1 + x_2 = \pi$$
$$x_2 = \pi - x_1$$

(1) Ničle: $a = 0$ $\sin x = 0$

$$x = 0^0 + k\Pi \qquad k \in \mathbb{Z} \qquad \mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$$

(2) Maksimume: $a = 1$ $\sin x = 1$ $x = \frac{\Pi}{2} + 2k\Pi$ $k \in \mathbb{Z}$

(3) Minimume: $a = -1$ $\sin x = -1$ $x = \frac{3\Pi}{2} + 2k\Pi$ $k \in \mathbb{Z}$

(4) Za poljubno vrednost: $-1 \le a \le 1$ $\sin x = a$

$$x_1 = \arcsin a + 2k\pi$$
$$x_2 = \pi - x_1 + 2k\pi \qquad k \in \mathbb{Z}$$

Na splošno velja formula (4), vandar za $a = 0,\ a = -1,\ a = 1$ raje računam po (1), (2), (3).

3.a) $\sin x = 0$ (1)

$$x = 0 + k\pi \qquad k \in \mathbb{Z}$$

3.f) $\sin x = \frac{\sqrt{3}}{2}$ (4)

$$x_1 = \arcsin\frac{\sqrt{3}}{2} + 2k\pi \qquad k \in \mathbb{Z}$$

$$x_1 = \frac{\pi}{3} + 2k\pi$$

$$x_2 = \pi - \frac{\pi}{3} + 2k\pi$$

$$x_2 = \frac{2\pi}{3} + 2k\pi$$

3.g) $\sin x = 1{,}5$

Ker je $\sin x > 1$, enačba nima rešitve.

4.a) $\sin x = \frac{1}{2}$ (4)

$$x_1 = \arcsin\frac{1}{2} + 2k\pi$$

$$x_1 = \frac{\pi}{6} + k\pi \qquad k \in \mathbb{Z}$$

$$x_2 = \pi - \frac{\pi}{6} + 2k\pi$$

$$x_2 = \frac{5\pi}{6} + 2k\pi$$

4c) $\sin 2x = 0{,}173$ (4)

$$(2x)_1 = \arcsin 0{,}173 + 2k\pi$$

$$2x = 9{,}96^0 + k360^0 \;/:2$$

$$x_1 = 4{,}98 + k360^0 = 4^0 59' + k180^0$$

$$(2x)_2 = 180^0 - 9{,}96^0 + k360^0$$

$x_2 = 85{,}02^0 + k180^0$ (do stopinje natančno)

$x_2 = 58^0 1' + k180^0$ (do minute natančno)

5.č) $\sin(2x-1) = \frac{1}{2}$ (4)

$$(2x-1)_1 = \arcsin\frac{1}{2} + 2k\pi$$

$$(2x-1) = \frac{\pi}{6} + 2k\pi \;/.6$$

$$12x - 6 = \pi + 12k\pi$$

$$12x = \pi + 6 + 12k\pi \;/:12$$

$$x_1 = \frac{\pi + 6}{12} + k\pi \quad = 0{,}76 + k\pi$$

Rezultat je v radianih.

$$(2x-1)_2 = \pi - \arcsin\frac{1}{2} + 2k\pi$$

$$2x - 1 = \pi - \frac{\pi}{6} + 2k\pi \;/.6$$

$$12x - 6 = 6\pi - \pi + 12k\pi$$

$$12x = 5\pi + 6 + 12k\pi \;/:12$$

$$x_2 = \frac{5\pi + 6}{12} + k\pi \quad = 1{,}81 + k\pi$$

Rezultat je v radianih.

5.d) $\sin(x+3)=0{,}731$ (4)

$$(x+3)_1 = \arcsin 0{,}731 + 2k\pi \qquad k \in \mathbb{Z}$$

$$\underline{x_1 = \arcsin 0{,}731 - 3 + 2k\pi}$$

$$\underline{x_2 = \pi - \arcsin 0{,}731 - 3 + 2k\pi}$$

5.f) $\sin(x-90^0)+\sin(x-180^0)=0$

Opomba:

Uporabim formulo za faktorizacijo:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$2\sin\frac{x-90^0+x-180^0}{2}.\cos\frac{x-90^0-(x-180^0)}{2}=0\,/:2$$

$$\sin\frac{2x-270}{2}.\cos\frac{x-90^0-x+180^0}{2}=0$$

$$\sin\frac{2(x-135^0)}{2}.\cos\frac{90^0}{2}=0$$

$$\sin(x-135^0)\cos 45^0=0$$

$$\sin(x-135^0)=0$$

$$x-135^0=0^0+k\pi \qquad k \in \mathbb{Z}$$

$$\underline{x=\frac{3\pi}{4}+k\pi}$$

6.d) $\sin^2 x - \cos^2 x = \frac{1}{2}$

Uporabim formulo:

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

da dobim isto kotno funkcijo.

$$\sin^2 x - \cos^2 x = \frac{1}{2}$$

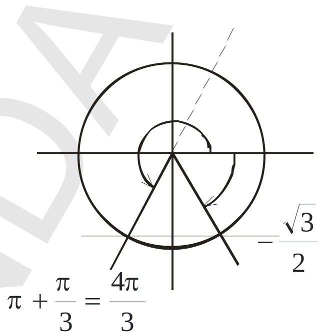
$$\sin^2 x - (1 - \sin^2 x) = \frac{1}{2}$$

$$\sin^2 x - 1 + \sin^2 x = \frac{1}{2}$$

$$2\sin^2 x - 1 = \frac{1}{2} / .2$$

$$4\sin^2 x - 3 = 0$$

$$(2\sin x - \sqrt{3})(2\sin x + \sqrt{3}) = 0$$



$2\sin x - \sqrt{3} = 0$ (4)

$$2\sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$x_1 = \arcsin\frac{\sqrt{3}}{2} + 2k\pi, \quad k \in \mathbb{Z}$$

$$\underline{x_1 = \frac{\pi}{3} + 2k\pi}$$

$$\underline{x_2 = \frac{2\pi}{3} + 2k\pi}$$

$2\sin x + \sqrt{3} = 0$ (4)

$$2\sin x = -\sqrt{3}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x_3 = \arcsin(-\frac{\sqrt{3}}{2}) + 2k\pi \quad k \in \mathbb{Z}$$

$$\underline{x_3 = \frac{4\pi}{3} + 2k\pi}$$

$$x_4 = \pi - x_3 + 2k\pi$$

$$x_4 = \pi - \frac{4\pi}{3} + 2k\pi$$

$$\underline{x_4 = -\frac{\pi}{3} + 2k\pi}$$

7.a) $\sin^2 x - 5\sin x - 6 = 0$

Vpeljem novo neznanko

$$\sin x = t$$

$$t^2 - 5t - 6 = 0$$

$$(t+1)(t-6) = 0$$

$t + 1 = 0$ $\quad$ $t - 6 = 0$

$t_1 = -1$ $\quad$ $t_2 = 6$

$\sin x = -1$ (2)

$$x = -\frac{\pi}{2} + 2k\pi$$

$k \in \mathbb{Z}$

$\sin x = 6 > 1$

Zato nimam rešitve